Resilience of Orbital Inspections to Partial Loss of Control Authority of the Chaser Satellite

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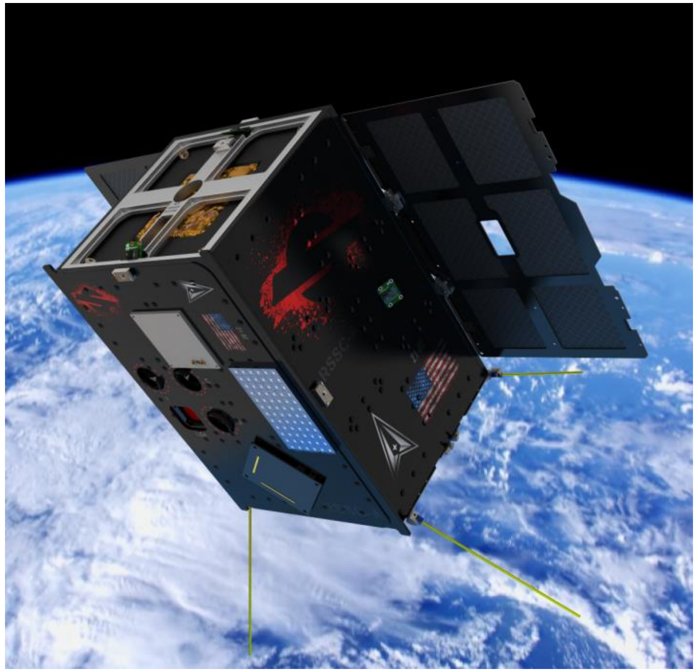
Satellite inspection and loss of control authority



Laura, an Orbital Robot from Rogue dedicated to satellite inspection.

The Nauka module of the ISS lost control authority over one of its thrusters.

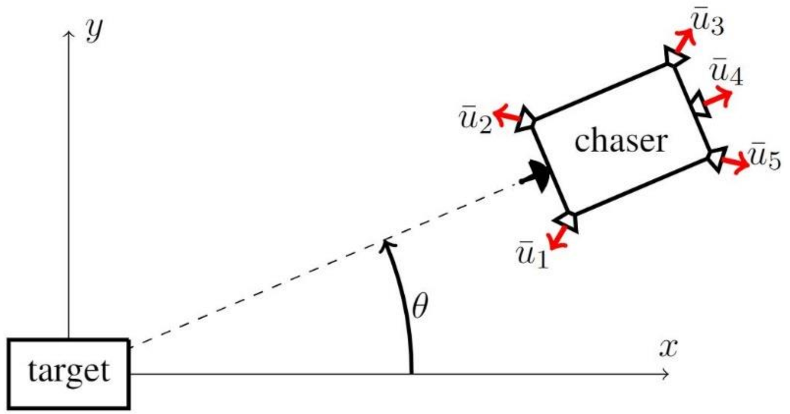
Rogue Space Systems Corporation, https://rogue.space/.
M. Bartels, "Russia says 'software failure' caused thruster misfire at space station," space.com, 2021, https://www.space.com/space-station-nauka-arrival-thruster-fire-update.

Clohessy-Wiltshire dynamics

$$\dot{X}(t) = AX(t) + rR_\theta \bar{B}\bar{u}(t), \qquad X = \begin{bmatrix} x \\ y \\ \dot{x} \\ \dot{y} \end{bmatrix}, \qquad X(0) = X_0, \qquad \bar{u}_i(t) \in [0,1]$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3\Omega^2 & 0 & 0 & 2\Omega \\ 0 & 0 & -2\Omega & 0 \end{bmatrix} \qquad \bar{B} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -\sqrt{2} & -1 \\ 1 & -1 & -1 & 0 & 1 \end{bmatrix}$$

Ω mean orbital rate, r thrust-to-weight ratio, R_θ rotation matrix.



Relative positions and attitude of both satellites, with the camera of the chaser always pointing at the target.

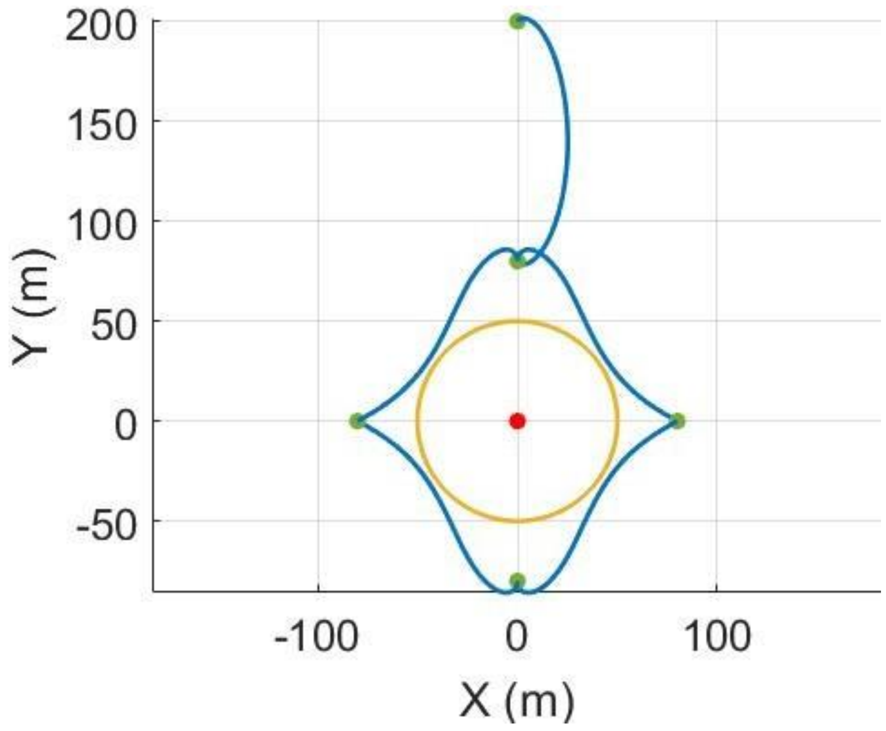
Loss of control authority over thruster 4 and actuation delay τ

$$\dot{X}(t) = AX(t) + rR_\theta B u(t, X(t-\tau), w(t-\tau)) + rR_\theta C w(t),$$

$$X(0) = X_0, \qquad u_i(t) \in [0,1], \qquad w(t) \in [0,1]$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix} \qquad C = \begin{bmatrix} 0 \\ 0 \\ -\sqrt{2} \\ 0 \end{bmatrix}$$

Initial trajectory



- Target satellite (red)
- Four holding points at 80m (green)
- Keep-out sphere (KOS) of radius 50m (yellow)
- Maximal velocity during inspection 5cm/s
- Fuel optimal trajectory (blue)

M. Vavrina et al., “Safe rendezvous trajectory design for the Restore-L mission,” 29th AAS/AIAA Space Flight Mechanics Meeting, 2019.
C. Jewison and R. S. Erwin, “A spacecraft benchmark problem for hybrid control and estimation,” 55th IEEE Conference on Decision and Control, 2016.
N. Ortolano et al., “Autonomous optimal trajectory planning for orbital rendezvous, satellite inspection, and final approach based on convex optimization,” Journal of the Astronautical Sciences, 2021

Spacecraft resilience

Resilience: for all targets X_{goal} and all undesirable thrust signal $w(t) \in \mathcal{W}$ there exists a control signal $u_w(t) \in \mathcal{U}$ such that the malfunctioning system (1) can reach X_{goal}.

controlled inputs

undesirable inputs

(1) $$\begin{cases} \dot{X}(t) = AX(t) + rR_\theta Bu(t) + rR_\theta Cw(t), \\ X(0) = X_0, \quad u(t) \in \mathcal{U}, \quad w(t) \in \mathcal{W}. \end{cases}$$

effective control inputs

(2) $$\begin{cases} \dot{X}(t) = AX(t) + rR_\theta p(t), \\ X(0) = X_0, \quad p(t) \in \mathcal{P} \end{cases}$$

Hájek's duality theorem: system (1) is resilient if and only if system (2) is controllable.

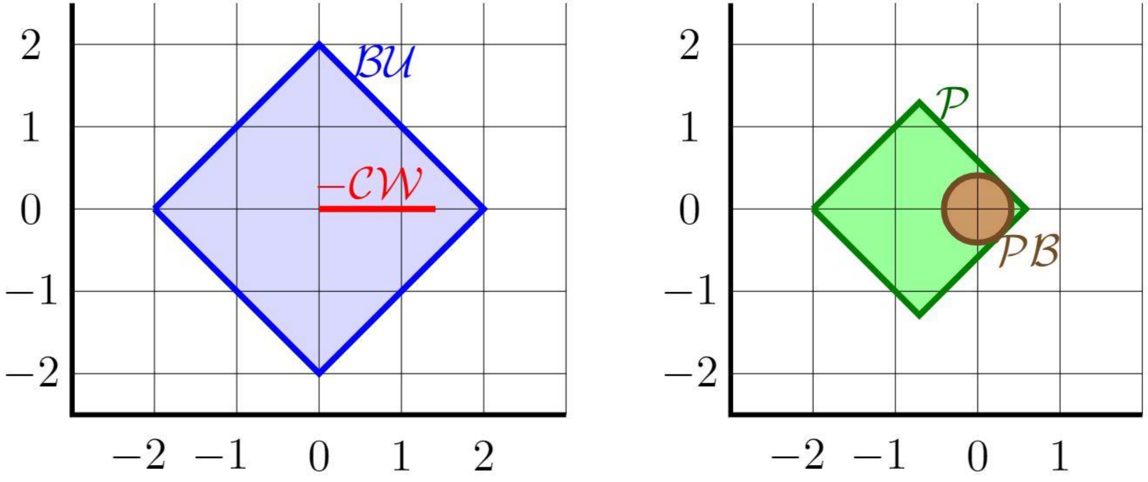
Minkowski difference of input sets:

$$\mathcal{P} = B\mathcal{U} \ominus (-C\mathcal{W}) = \{p \in B\mathcal{U} : p - Cw \in B\mathcal{U} \text{ for all } w \in \mathcal{W}\}$$

$$B\mathcal{U} = \{Bu : u \in \mathcal{U}\} \quad \text{and} \quad C\mathcal{W} = \{Cw : w \in \mathcal{W}\}.$$

O. Hájek, "Duality for differential games and optimal control," Mathematical Systems Theory, 1974.
J.-B. Bouvier and M. Ornik, "Quantitative Resilience of Linear Systems," 20th European Control Conference, 2022.

Spacecraft resilience



Polygons $\mathcal{BU}$ (blue), $-\mathcal{CW}$ (red), their Minkowski difference $\mathcal{P}$ (green) and the largest ball $\mathcal{PB}$ (brown) centered on 0 in $\mathcal{P}$.

$$\mathcal{P} = B\mathcal{U} \ominus (-C\mathcal{W}) = \{p \in B\mathcal{U} : p - Cw \in B\mathcal{U} \;\; for\, all\, w \in \mathcal{W}\}.$$

$$(2) \quad \begin{cases} \dot{X}(t) = AX(t) + rR_\theta p(t), \\ X(0) = X_0, \quad p(t) \in \mathcal{P} \end{cases}$$

System (2) is nonlinear because of the rotation R_θ, which complicates the controllability verification.

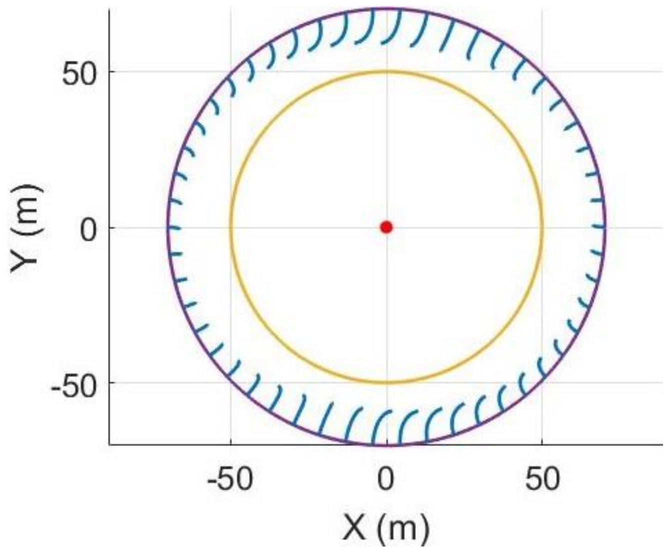
$$(3) \quad \begin{cases} \dot{X}(t) = AX(t) + r\hat{B}p(t), \\ X(0) = X_0, \quad p(t) \in \mathcal{PB}. \end{cases}$$

System (3) is controllable, hence so is (2).
Then the spacecraft is resilient to the loss of control authority over thruster 4.

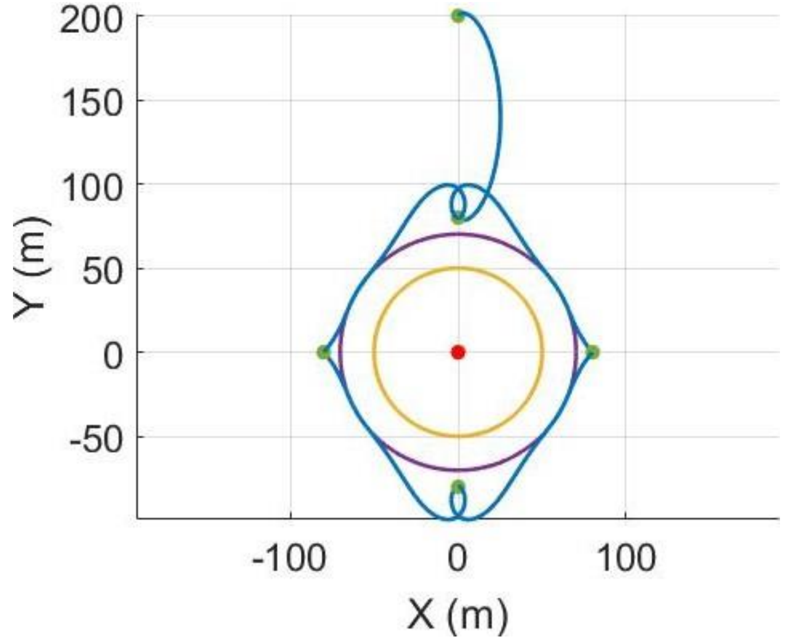
R. F. Brammer, "Controllability in linear autonomous systems with positive controllers," SIAM Journal on Control, 1972.

Safe reference trajectory

Resilience does not handle actuation delay and trajectory tracking, so instead we calculate the minimal increase of the keep-out sphere (KOS) radius guaranteeing safety.

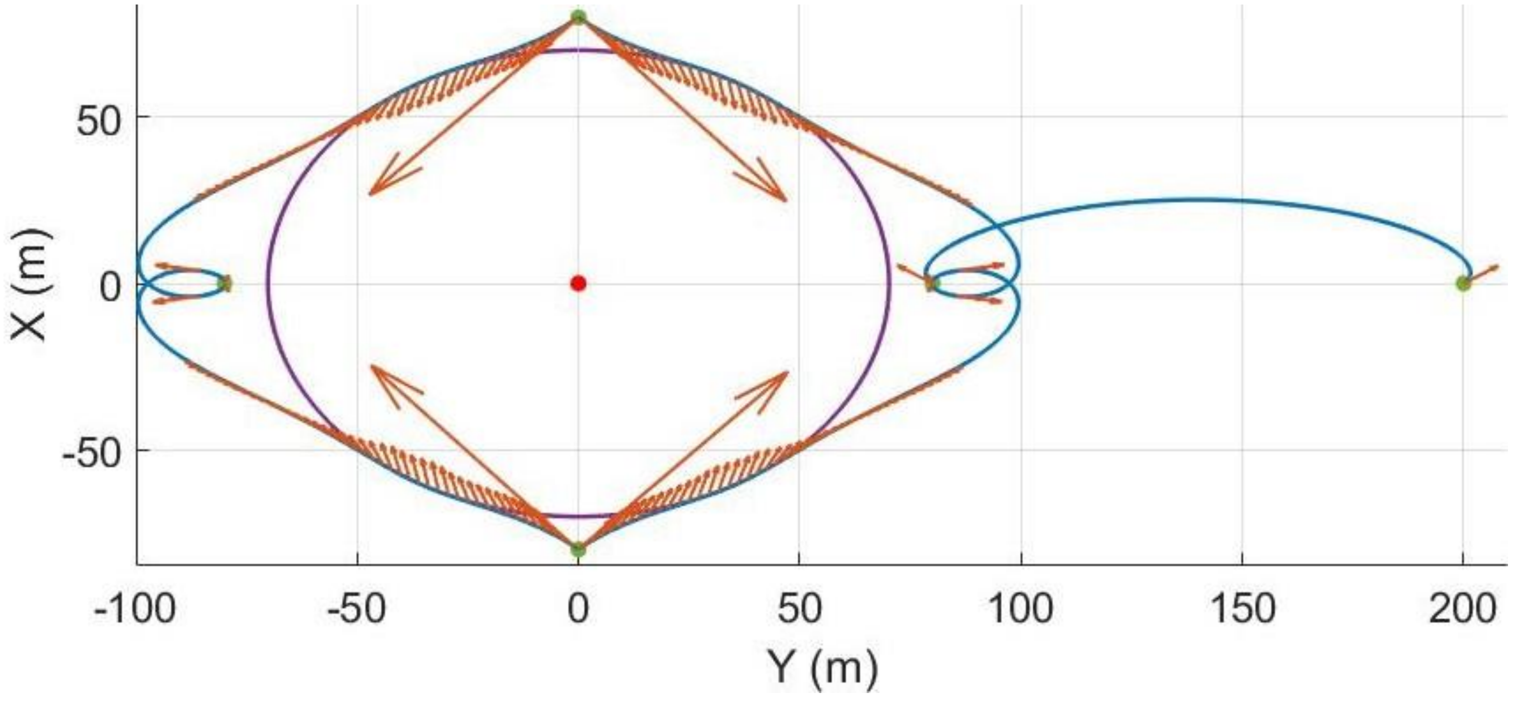


All worst-case trajectories (blue) starting from the increased KOS (purple) with initial velocity V_{max} never reach the initial KOS (yellow).

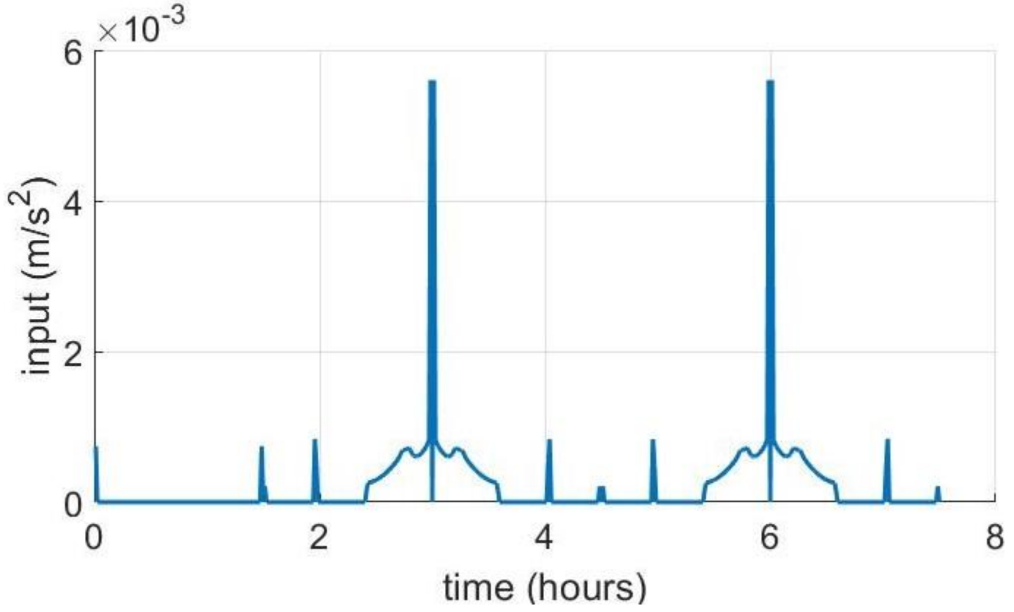


Reference minimal-fuel trajectory (blue) with 5 waypoints (green) to inspect the target satellite (red) without breaching the increased KOS (purple).

Analysis of the reference trajectory



Reference trajectory (blue) with thrust impulses (red arrows).



Magnitude of the reference input signal $||u||$.

Controller design

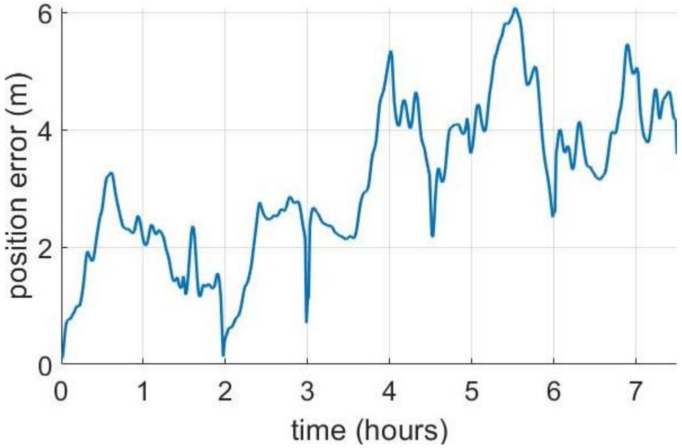
To estimate $X(t+\tau)$ based on data available at time t, we use the Léchappé state predictor

$$X_p(t) = e^{A\tau}X(t) + \int_{t-\tau}^{t} e^{A(t-s)} r R_\theta \big(Bu(s) + Cw(s)\big)\, ds.$$

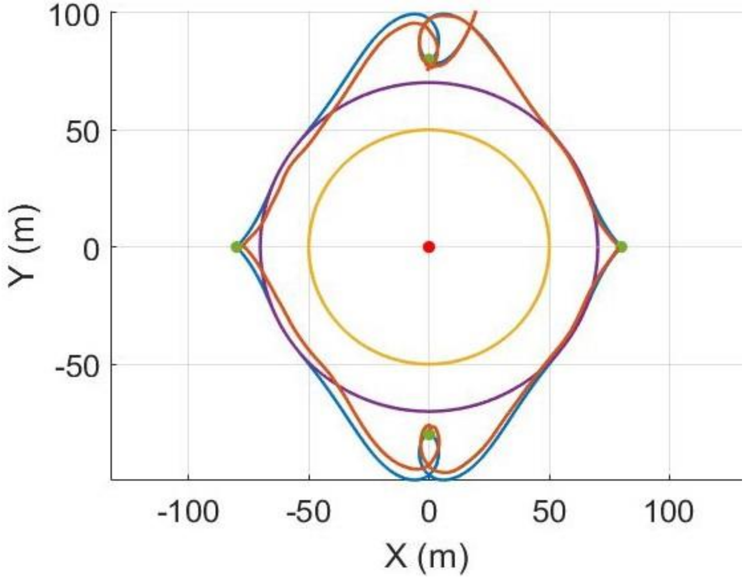
Adaptive trajectory tracking

$$u(t) = u_{ref}(t) + K_\theta\big(X_{ref}(t) - X_p(t)\big).$$

Mediocre tracking performance:



Ever growing position error of the tracking trajectory.



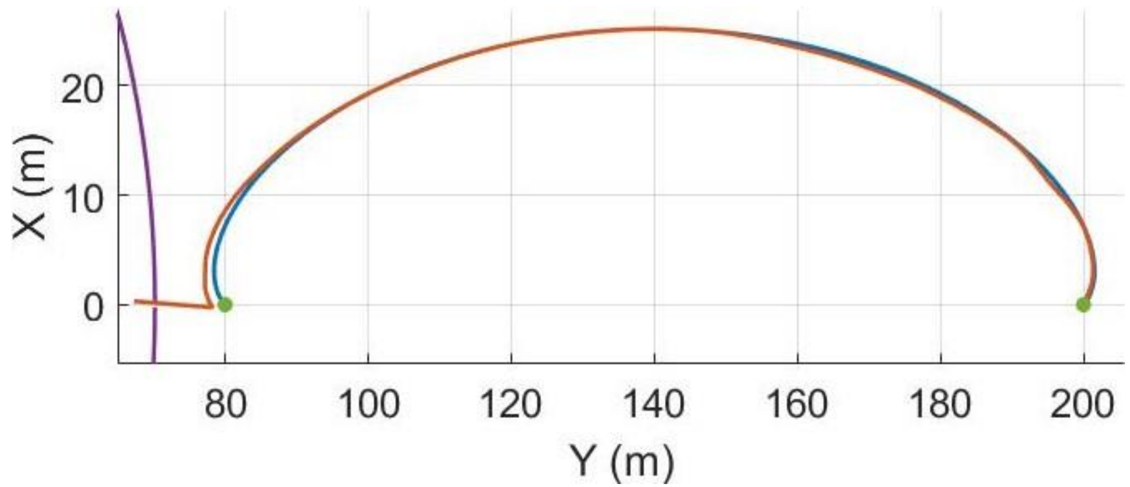
Tracking trajectory (red) moving further away from the reference (blue).

V. Léchappé et al., “New predictive scheme for the control of LTI systems with input delay and unknown disturbances,” Automatica, 2015.

D. Bresch-Pietri and M. Krstic, “Adaptive trajectory tracking despite unknown input delay and plant parameters,” Automatica, 2009.

Controller design

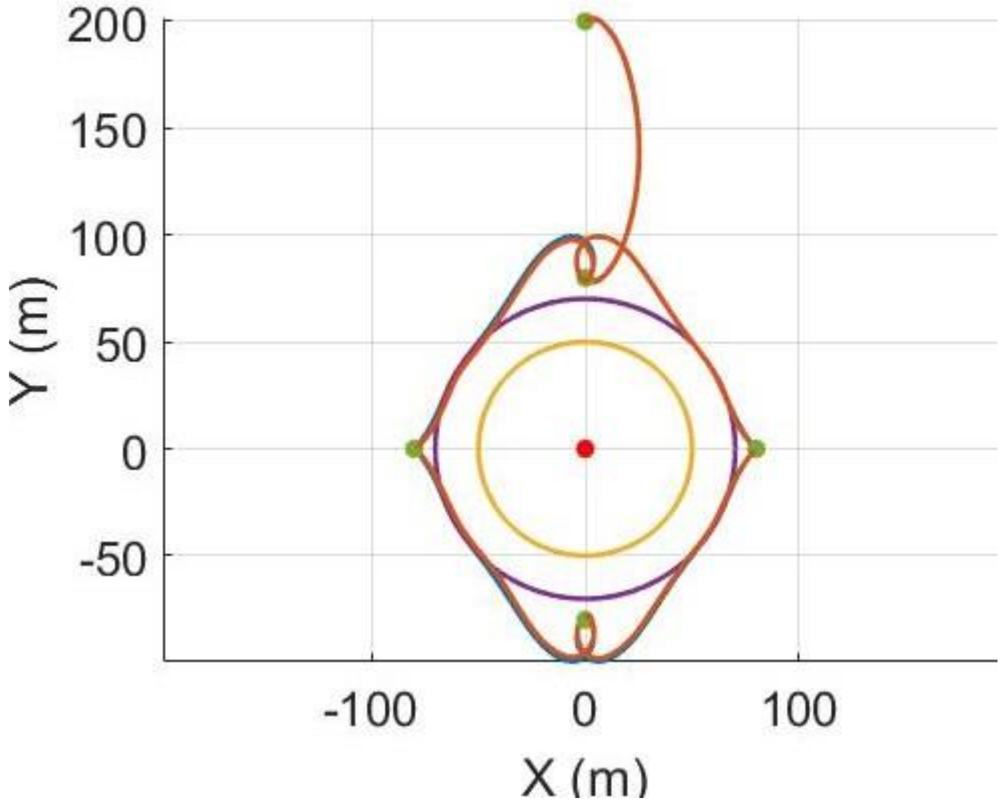
Upgrade controller to PID $u(t) = u_{ref}(t) + K_\theta \left(k_p \Delta(t) + k_i \int_0^t \Delta(s) ds + k_d \frac{d}{dt} \Delta(t) \right)$, with $\Delta(t) = X_{ref}(t) - X_p(t)$.



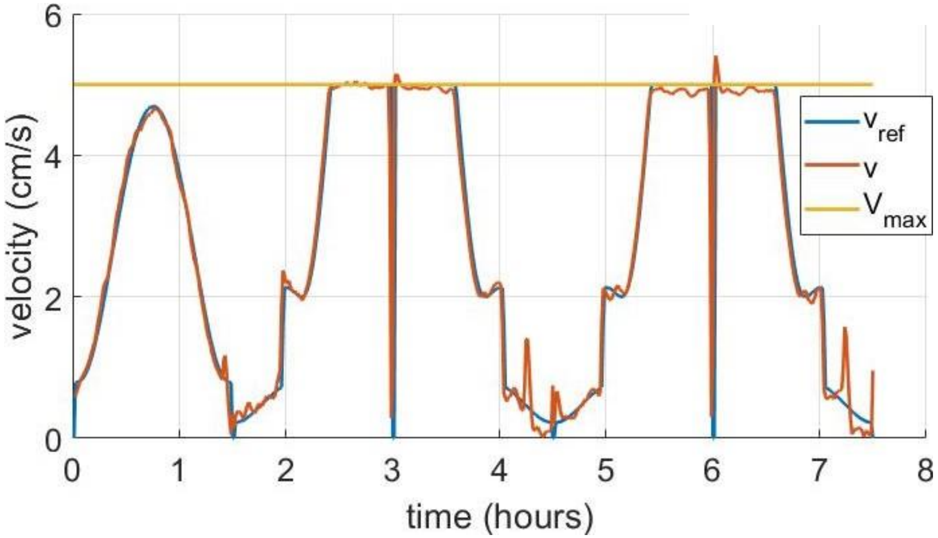
Tracking trajectory (red) does not converge to the left waypoint (green) despite a better tracking of the reference (blue).

We switch the tracking error $\Delta(t)$ to $\Delta_g(t) = X_{goal} - X_p(t)$ when close enough from waypoint X_{goal}.

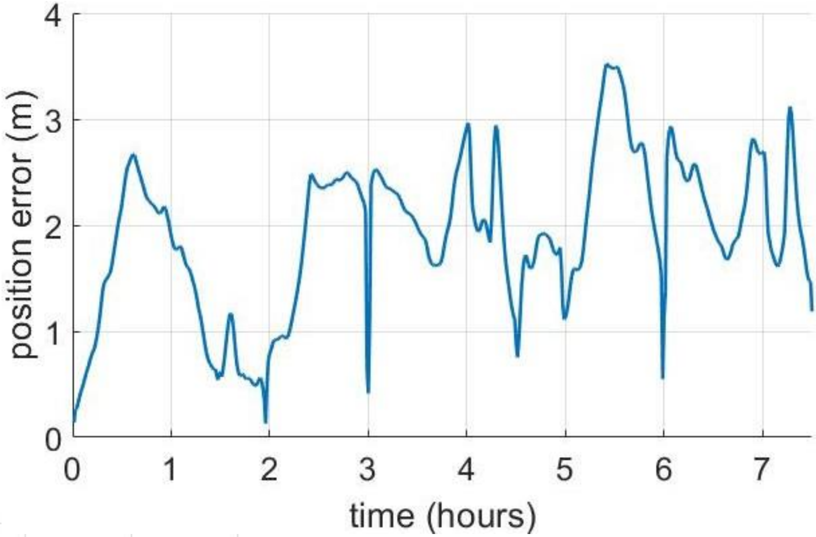
Numerical simulation



Trajectory tracking (red) of the reference (blue).

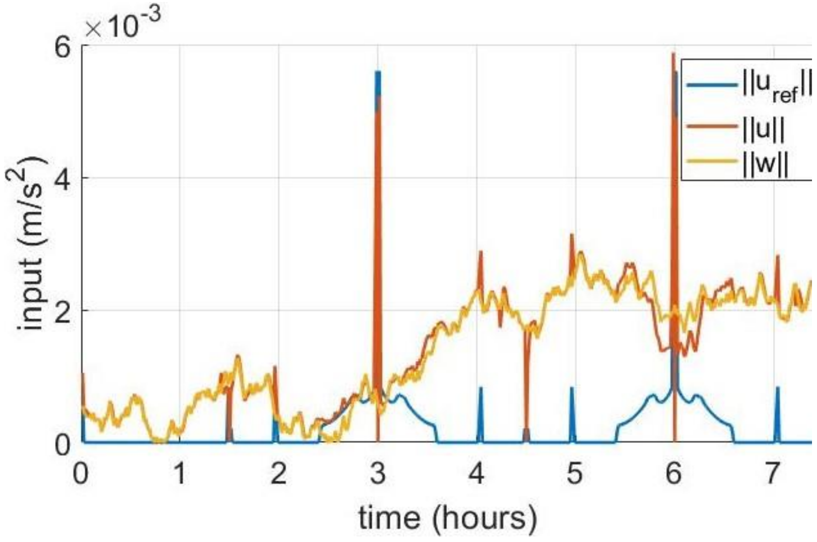


Comparison of the reference velocity (blue) with the tracking velocity (red) and the maximal velocity (yellow).

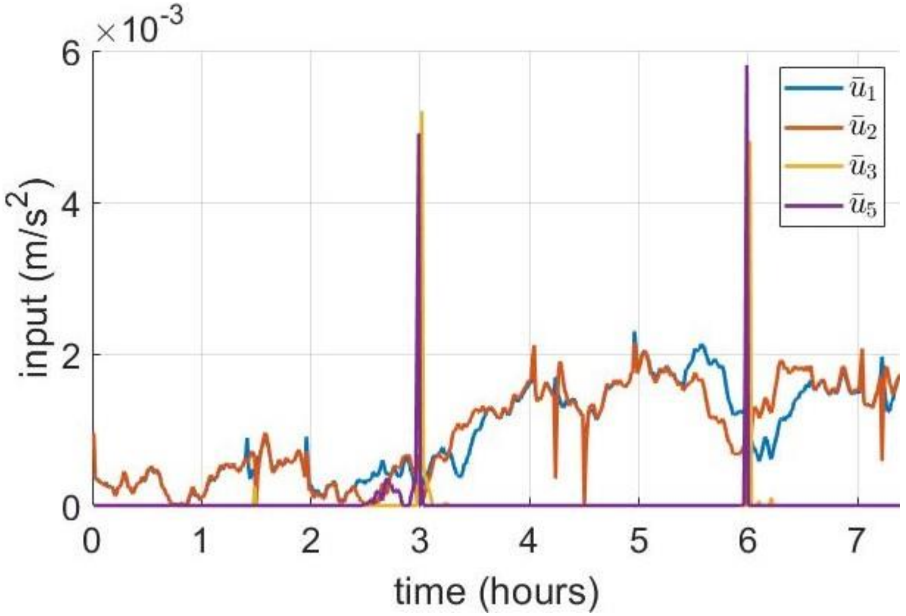


Position error between the tracking trajectory and the reference.

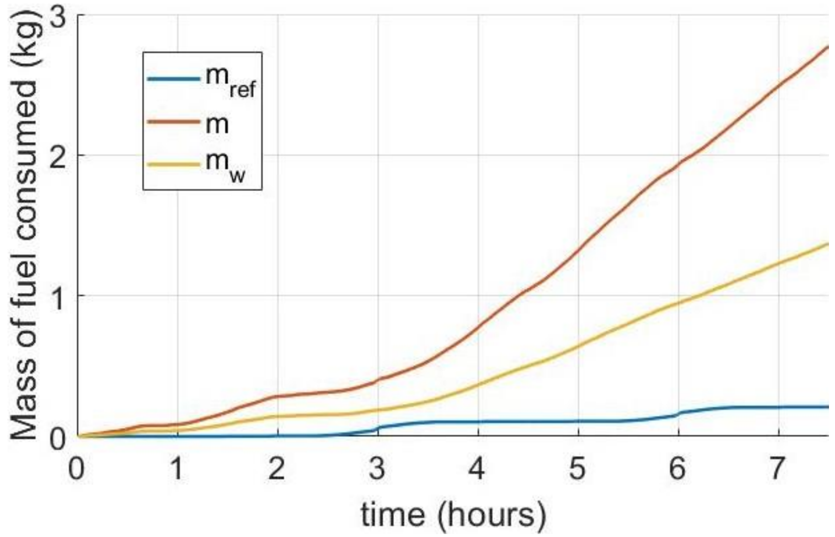
Trajectory analysis



Thrust magnitude on the tracking trajectory (red), the reference (blue), and the undesirable thrust (yellow).

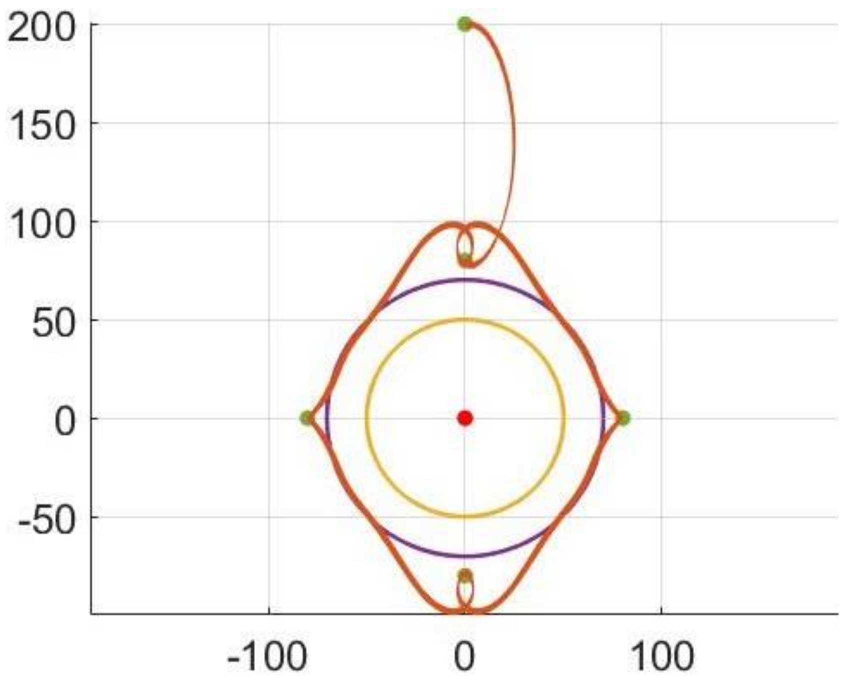


Thrust profiles of the four controlled thrusters.

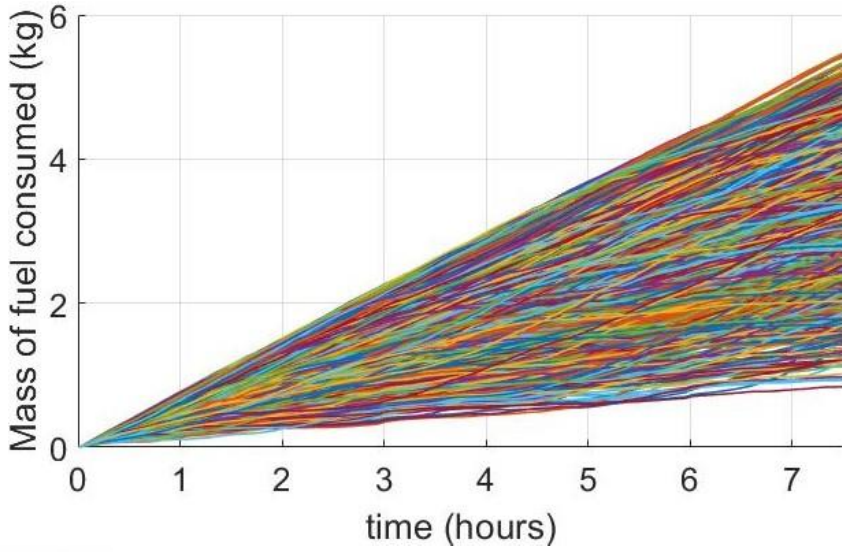


Mass of fuel consumed on the tracking trajectory (red), on the reference (blue) and by the malfunctioning thruster (yellow).

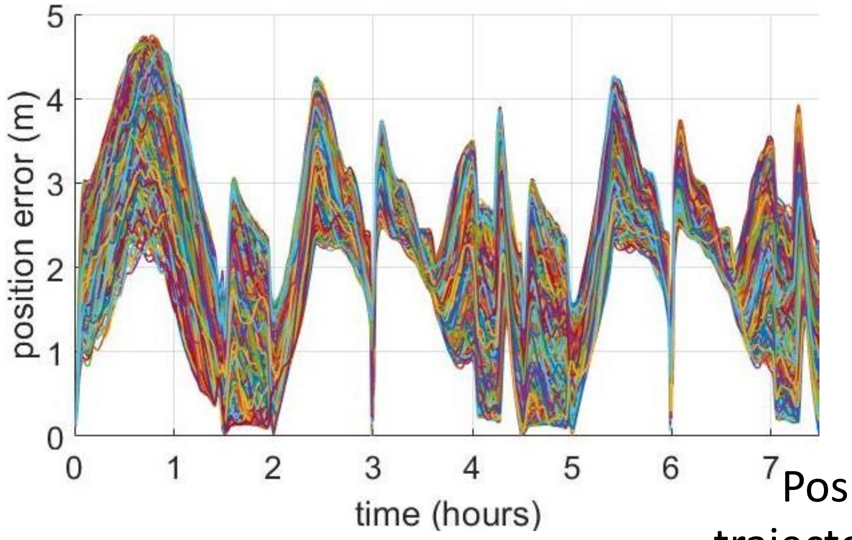
Monte Carlo simulation



1000 tracking trajectories (red).

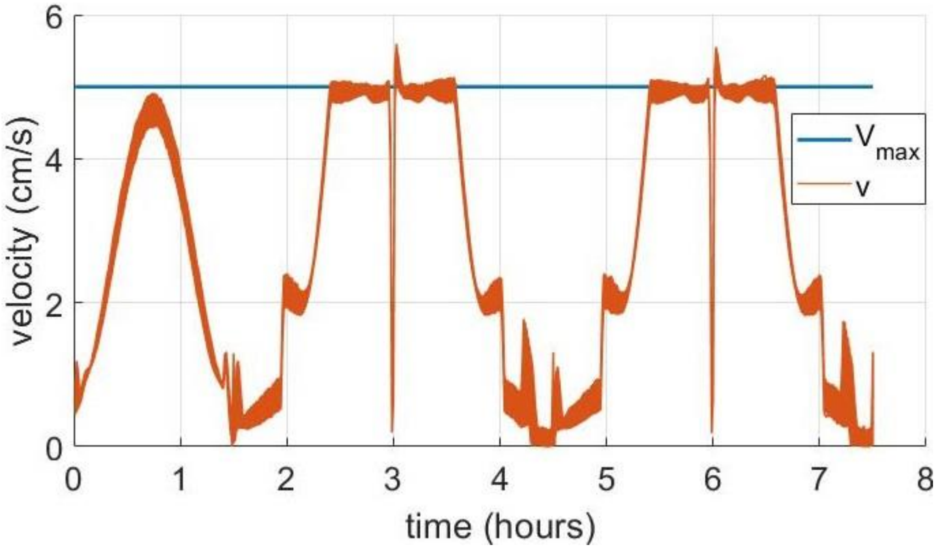


Fuel consumption of the tracking trajectories.

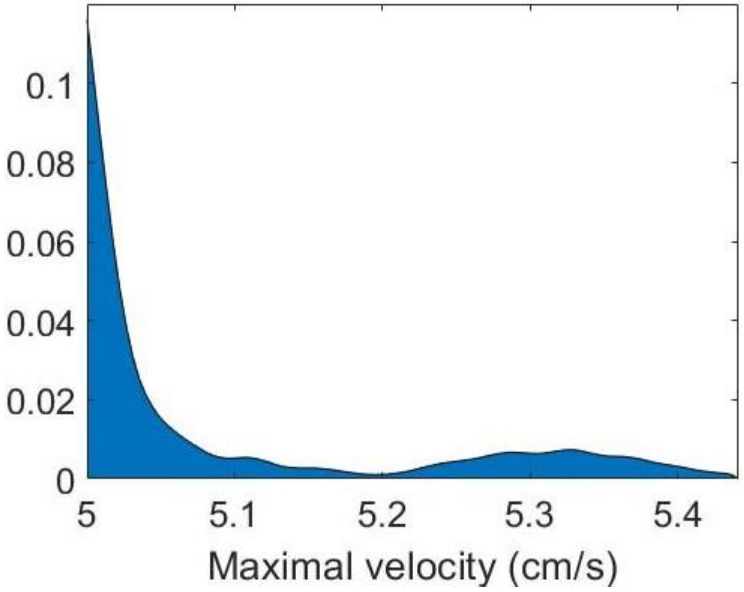


Position error between the reference trajectory and multiple tracking trajectories.

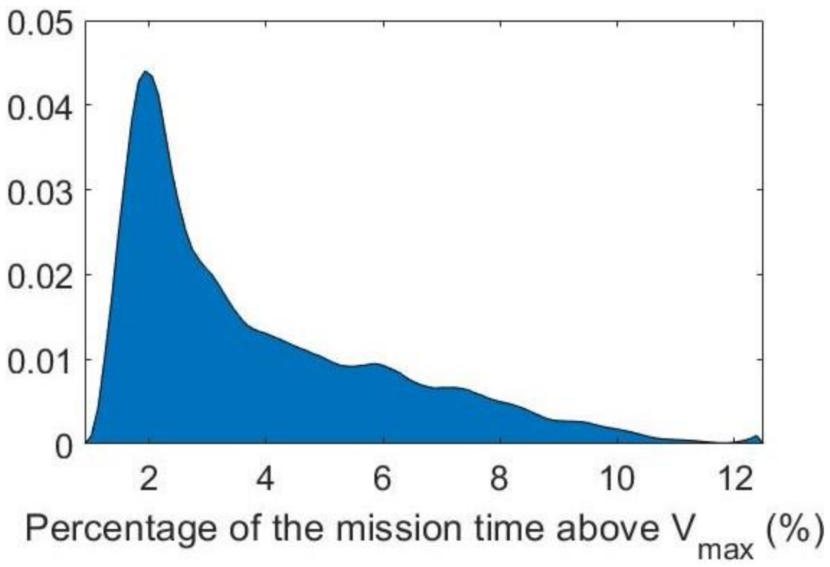
Monte Carlo simulation



Velocity profile of the multiple tracking trajectories.



Probability distribution of the maximal velocity of the tracking trajectory.



Probability distribution of the percentage of the mission time spent at velocities higher than V_{max}.

Conclusion

We used predictors, PID control and resilience theory to ensure the safety of a satellite inspection mission despite actuation delays and the loss of control authority over a thruster.

Thank you for your attention.